2-6 Special Functions

Main Ideas

- Identify and graph step, constant, and identity functions.
- Identify and graph absolute value and piecewise functions.

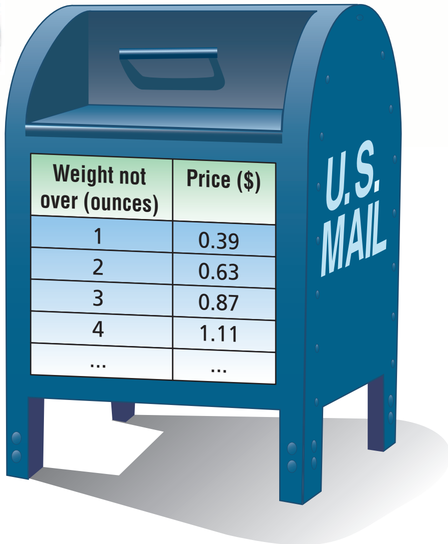
New Vocabulary

step function
greatest integer function
constant function
identity function
absolute value function
piecewise function

GET READY for the Lesson

The cost of the postage to mail a letter is a function of the weight of the letter. But the function is not linear. It is a special function called a **step function**.

For letters with weights between whole numbers, the cost "steps up" to the next higher cost. So the cost to mail a 1.5-ounce letter is the same as the cost to mail a 2-ounce letter, \$0.63.



Weight not over (ounces)	Price (\$)
1	0.39
2	0.63
3	0.87
4	1.11
...	...

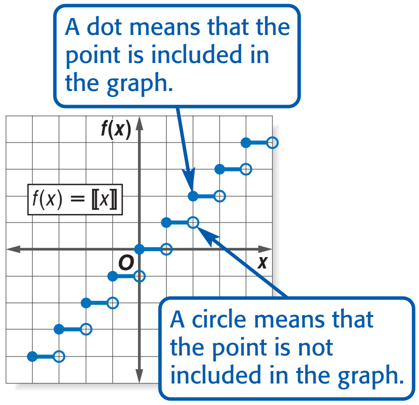
Step Functions, Constant Functions, and the Identity Function The graph of a step function is not linear. It consists of line segments or rays. The **greatest integer function**, written $f(x) = [\![x]\!]$, is an example of a step function. The symbol $[\![x]\!]$ means *the greatest integer less than or equal to x.* For example, $[\![7.3]\!] = 7$ and $[\![-1.5]\!] = -2$ because $-1 > -1.5$.

Study Tip

Greatest Integer Function

Notice that the domain of this step function is all real numbers and the range is all integers.

$f(x) = [\![x]\!]$	
x	$f(x)$
$-3 \le x < -2$	-3
$-2 \le x < -1$	-2
$-1 \le x < 0$	-1
$0 \le x < 1$	0
$1 \le x < 2$	1
$2 \le x < 3$	2
$3 \le x < 4$	3



Real-World EXAMPLE Step Function

1 **BUSINESS** **The No Leak Plumbing Repair Company charges \$60 per hour or any fraction thereof for labor. Draw a graph that represents this situation.**

Explore The total labor charge must be a multiple of \$60, so the graph will be the graph of a step function.

Plan If the time spent on labor is greater than 0 hours, but less than or equal to 1 hour, then the labor cost is \$60. If the time is greater than 1 hour but less than or equal to 2 hours, then the labor cost is \$120, and so on.

(continued on the next page)

Solve Use the pattern of times and costs to make a table, where x is the number of hours of labor and $C(x)$ is the total labor cost. Then graph.

Concepts in Motion
Animation
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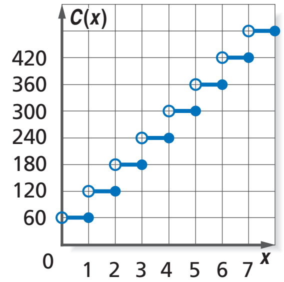
x	$C(x)$
$0 < x \le 1$	\$60
$1 < x \le 2$	\$120
$2 < x \le 3$	\$180
$3 < x \le 4$	\$240
$4 < x \le 5$	\$300



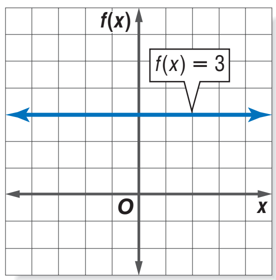
Check Since the company rounds any fraction of an hour up to the next whole number, each segment on the graph has a circle at the left endpoint and a dot at the right endpoint.

CHECK Your Progress

1. RECYCLING A recycling company pays \$5 for every full box of newspaper. They do not give any money for partial boxes. Draw a graph that shows the amount of money for the number of boxes brought to the center.

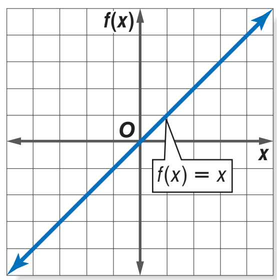
You learned in Lesson 2-4 that the slope-intercept form of a linear function is $y = mx + b$, or in function notation, $f(x) = mx + b$.

When $m = 0$, the value of the function is $f(x) = b$ for every x-value. So, $f(x) = b$ is called a **constant function**. *The function $f(x) = 0$ is called the zero function.*



Another special case of slope-intercept form is $m = 1$, $b = 0$. This is the function $f(x) = x$. The graph is the line through the origin with slope 1.

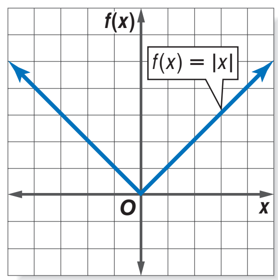
Since the function does not change the input value, $f(x) = x$ is called the **identity function**.



Absolute Value and Piecewise Functions

Another special function is the **absolute value function**, $f(x) = |x|$.

$f(x) = \|x\|$	
x	$f(x)$
-2	2
-1	1
0	0
1	1
2	2



Study Tip

Absolute Value Function

Notice that the domain is all real numbers and the range is all nonnegative real numbers.

The absolute value function can be written as $f(x) = \begin{cases} -x \text{ if } x < 0 \\ x \text{ if } x \geq 0 \end{cases}$. A function that is written using two or more expressions is called a **piecewise function.**

Recall that a family of graphs displays one or more similar characteristics. The parent graph of most absolute value functions is $y = |x|$.

EXAMPLE Absolute Value Functions

2 **Graph $f(x) = |x| + 1$ and $g(x) = |x| - 2$ on the same coordinate plane. Determine the similarities and differences in the two graphs.**

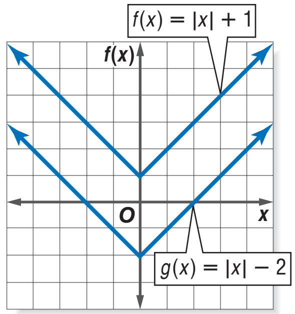
Find several ordered pairs for each function.

x	$\|x\| + 1$
−2	3
−1	2
0	1
1	2

x	$\|x\| - 2$
−2	0
−1	−1
0	−2
1	−1

Graph the points and connect them.

- The domain of each function is all real numbers.
- The range of $f(x) = |x| + 1$ is $\{y \mid y \geq 1\}$.
  The range of $g(x) = |x| - 2$ is $\{y \mid y \geq -2\}$.
- The graphs have the same shape, but different y–intercepts.
- The graph of $g(x) = |x| - 2$ is the graph of $f(x) = |x| + 1$ translated down 3 units.



CHECK Your Progress

2. Graph $f(x) = |x + 1|$ and $g(x) = |x - 2|$.

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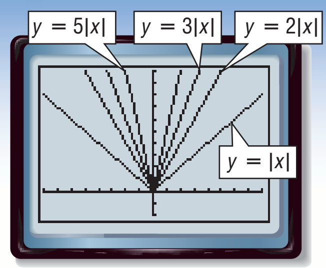
You can also use a graphing calculator to investigate families of absolute value graphs.

GRAPHING CALCULATOR LAB

Family of Absolute Value Graphs

The calculator screen shows the graphs of $y = |x|$, $y = 2|x|$, $y = 3|x|$, and $y = 5|x|$.



[−8, 8] scl: 1 by [−2, 10] scl: 1

THINK AND DISCUSS

1. What do these graphs have in common?
2. Describe how the graph of $y = a|x|$ changes as a increases. Assume $a > 0$.
3. Write an absolute value function whose graph is between the graphs of $y = 2|x|$ and $y = 3|x|$.
4. Graph $y = |x|$ and $y = -|x|$ on the same screen. Then graph $y = 2|x|$ and $y = -2|x|$ on the same screen. What is true in each case?
5. In general, what is true about the graph of $y = a|x|$ when $a < 0$?

To graph other piecewise functions, examine the inequalities in the definition of the function to determine how much of each piece to include.

Study Tip

Graphs of Piecewise Functions

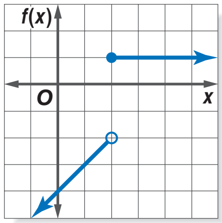
The graphs of each part of a piecewise function may or may not connect. A graph may stop at a given x value and then begin again at a different y value for the same x value.

EXAMPLE 3 Piecewise Function

Graph $f(x) = \begin{cases} x - 4 \text{ if } x < 2 \\ 1 \text{ if } x \geq 2 \end{cases}$. Identify the domain and range.

Step 1 Graph the linear function $f(x) = x - 4$ for $x < 2$. Since 2 does not satisfy this inequality, stop with an open circle at $(2, -2)$.

Step 2 Graph the constant function $f(x) = 1$ for $x \geq 2$. Since 2 does satisfy this inequality, begin with a closed circle at $(2, 1)$ and draw a horizontal ray to the right.



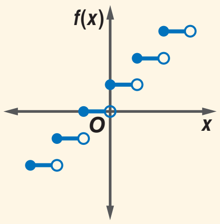
The function is defined for all values of x, so the domain is all real numbers. The values that are y-coordinates of points on the graph are 1 and all real numbers less than -2, so the range is $\{y \mid y < -2 \text{ or } y = 1\}$.

CHECK Your Progress

3. Graph $f(x) = \begin{cases} x + 2 \text{ if } x < 0 \\ x \text{ if } x \geq 0 \end{cases}$. Identify the domain and range.

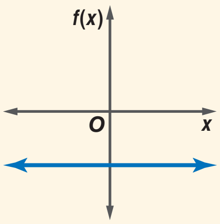
CONCEPT SUMMARY — *Special Functions*

Step Function



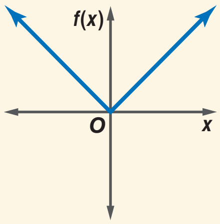
horizontal segments and/or rays

Constant Function



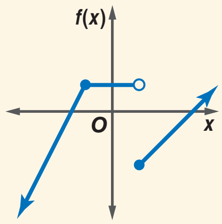
horizontal line

Absolute Value Function



V-shape

Piecewise Function

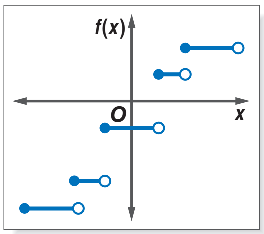


different rays, segments, and curves

EXAMPLE 4 Identify Functions

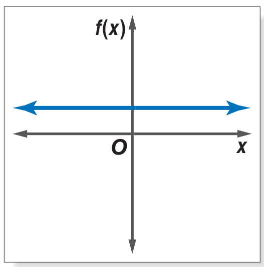
Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.

a.



The graph has multiple horizontal segments. It represents a step function.

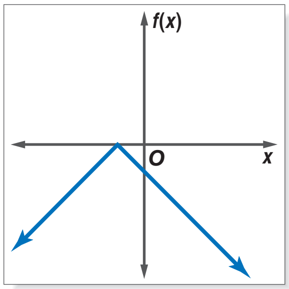
b.



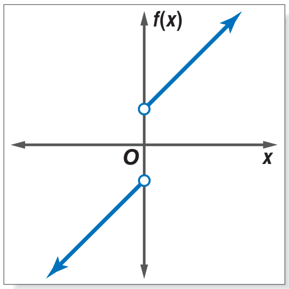
The graph is a horizontal line. It represents a constant function.

Check Your Progress

4A.



4B.



Check Your Understanding

Examples 1–3 (pp. 95–98)

Graph each function. Identify the domain and range.

1. $f(x) = -[\![x]\!]$ **2.** $g(x) = [\![2x]\!]$ **3.** $f(x) = 4$

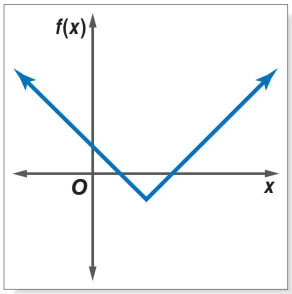
4. $z(x) = -3$ **5.** $h(x) = |x| - 3$ **6.** $f(x) = |3x - 2|$

7. $g(x) = \begin{cases} -1 \text{ if } x < 0 \\ -x + 2 \text{ if } x \geq 0 \end{cases}$ **8.** $h(x) = \begin{cases} x + 3 \text{ if } x \leq -1 \\ 2x \text{ if } x > -1 \end{cases}$

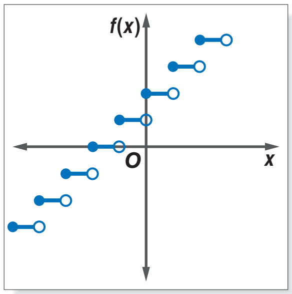
Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

Example 4 (pp. 98–99)

9.



10.



PARKING For Exercises 11–13, use the following information.
A downtown parking lot charges \$2 for the first hour and \$1 for each additional hour or part of an hour.

11. What type of special function models this situation?

12. Draw a graph of a function that represents this situation.

13. Use the graph to find the cost of parking there for $4\frac{1}{2}$ hours.

Exercises

HOMEWORK HELP

For Exercises	See Examples
14–19	1
20–25	2
26–27	3
28–33	4

Graph each function. Identify the domain and range.

14. $f(x) = [\![x + 3]\!]$ **15.** $g(x) = [\![x - 2]\!]$ **16.** $f(x) = 2[\![x]\!]$

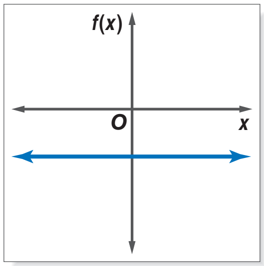
17. $h(x) = -3[\![x]\!]$ **18.** $g(x) = [\![x]\!] + 3$ **19.** $f(x) = [\![x]\!] - 1$

20. $f(x) = |2x|$ **21.** $h(x) = |-x|$ **22.** $g(x) = |x| + 3$

23. $g(x) = |x| - 4$ **24.** $h(x) = |x + 3|$ **25.** $f(x) = |x + 2|$

26. $f(x) = \begin{cases} -x \text{ if } x \leq 3 \\ 2 \text{ if } x > 3 \end{cases}$ **27.** $h(x) = \begin{cases} -1 \text{ if } x < -2 \\ 1 \text{ if } x > 2 \end{cases}$

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

28.


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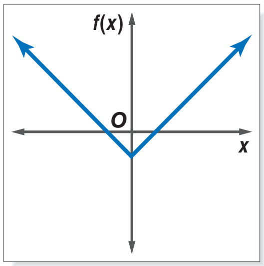

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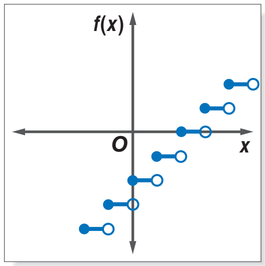

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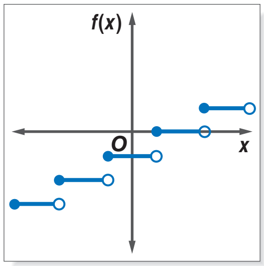

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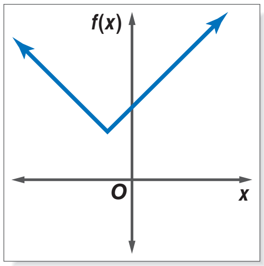

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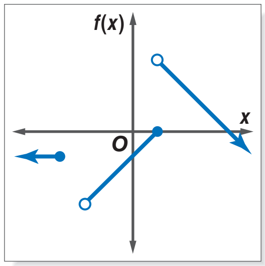

34. THEATER Springfield High School's theater can hold 250 students. The drama club is performing a play in the theater. Draw a graph of a step function that shows the relationship between the number of tickets sold x and the minimum number of performances y that the drama club must do.

Graph each function. Identify the domain and range.

35. $f(x) = \left|x - \frac{1}{4}\right|$

36. $f(x) = \left|x + \frac{1}{2}\right|$

37. $f(x) = \begin{cases} x \text{ if } x < -3 \\ 2 \text{ if } -3 \leq x < 1 \\ -2x + 2 \text{ if } x \geq 1 \end{cases}$

38. $g(x) = \begin{cases} -1 \text{ if } x \leq -2 \\ x \text{ if } -2 < x < 2 \\ -x + 1 \text{ if } x \geq 2 \end{cases}$

39. $f(x) = \llbracket\, |x| \,\rrbracket$

40. $g(x) = |\llbracket x \rrbracket|$

TELEPHONE RATES For Exercises 41 and 42, use the following information.
Masao has a long-distance telephone plan where she pays 10¢ for each minute or part of a minute that she talks, regardless of the time of day.

41. Graph a step function that represents this situation.

42. How much would a call that lasts 9 minutes and 40 seconds cost?

Real-World Link

Good sources of vitamin C include citrus fruits and juices, cantaloupe, broccoli, brussels sprouts, potatoes, sweet potatoes, tomatoes, and cabbage.

Source: *The World Almanac*

NUTRITION For Exercises 43–45, use the following information.
The recommended dietary allowance for vitamin C is 2 micrograms per day.

43. Write an absolute value function for the difference between the number of micrograms of vitamin C you ate today x and the recommended amount.

44. What is an appropriate domain for the function?

45. Use the domain to graph the function.

EXTRA PRACTICE
See pages 894, 927.
Math Online
Self-Check Quiz at algebra2.com

46. INSURANCE According to the terms of Lavon's insurance plan, he must pay the first $300 of his annual medical expenses. The insurance company pays 80% of the rest of his medical expenses. Write a function for how much the insurance company pays if x represents Lavon's annual medical expenses.

H.O.T. Problems

47. OPEN ENDED Write a function involving absolute value for which $f(-2) = 3$.

48. REASONING Find a counterexample to the statement *To find the greatest integer function of x when x is not an integer, round x to the nearest integer.*

49. CHALLENGE Graph $|x| + |y| = 3$.

50. ***Writing in Math*** Use the information on page 95 to explain how step functions apply to postage rates. Explain why a step function is the best model for this situation while your gas mileage as a function of time as you drive to the post office cannot be modeled with a step function. Then graph the function that represents the cost of a first-class letter.

STANDARDIZED TEST PRACTICE

51. ACT/SAT For which function does $f\left(-\frac{1}{2}\right) \neq -1$?

A $f(x) = 2x$

B $f(x) = |-2x|$

C $f(x) = [\![x]\!]$

D $f(x) = [\![2x]\!]$

52. ACT/SAT For which function is the range $\{y \mid y \leq 0\}$?

F $f(x) = -x$

G $f(x) = [\![x]\!]$

H $f(x) = |x|$

J $f(x) = -|x|$

53. REVIEW Solve: $5(x + 4) = x + 4$

Step 1: $5x + 20 = x + 4$

Step 2: $4x + 20 = 4$

Step 3: $4x = 24$

Step 4: $x = 6$

Which is the first *incorrrect* step in the solution shown above?

A Step 4

B Step 3

C Step 2

D Step 1

Spiral Review

HEALTH For Exercises 54–56, use the table that shows the life expectancy for people born in various years. (Lesson 2-5)

Year	1950	1960	1970	1980	1990	2000
Expectancy	68.2	69.7	70.8	73.7	75.4	77.0

Source: National Center for Health Statistics

54. Draw a scatter plot in which x is the number of years since 1940 and describe the correlation.

55. Find a prediction equation.

56. Predict the life expectancy of a person born in 2010.

Write an equation in slope-intercept form that satisfies each set of conditions. (Lesson 2-4)

57. slope 3, passes through $(-2, 4)$

58. passes through $(0, -2)$ and $(4, 2)$

Solve each inequality. Graph the solution set. (Lesson 1-3)

59. $3x - 5 \geq 4$

60. $28 - 6y < 23$

GET READY for the Next Lesson

PREREQUISITE SKILL Determine whether (0, 0) satisfies each inequality. Write *yes* or *no*. (Lesson 1-5)

61. $y < 2x + 3$

62. $y \geq -x + 1$

63. $y \leq \frac{3}{4}x - 5$

64. $2x + 6y + 3 > 0$

65. $y > |x|$

66. $|x| + y \leq 3$